Venture Capital Contracts with Informed Investors *

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Abstract

A cashless entrepreneur designs a contract backed by future cash flows to raise funds from an informed investor. The entrepreneur can exert costly effort to improve payoff relevant outcome where the marginal benefit depends on the quality of the project. The quality of the project is observed privately by the investors. This study investigates the optimal contract that an entrepreneur proposes to an investor given these two specific information frictions. In equilibrium, a trade-off emerges between the surplus maximization of projects of different qualities. In particular, this tradeoff creates an endogenous preference for risk by the entrepreneur when designing the menu of contracts. The results are applied to predict the effect of divergence in the entrepreneur's subjective beliefs on the project's outcome and overall efficiency in the economy.

JEL Classification: D82, D86, G24, G32, M13, L26, O31

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1 Introduction

Financial contracting drives innovation and employment growth by enabling investment in high-growth entrepreneurial firms. However, the financing of innovation is riddled with agency problems and asymmetric information which may prevent its efficient investment. Specifically, empirical evidence suggests that venture capital firms often have extensive experience and accumulated knowledge enabling them to judge entrepreneurial ventures better than the entrepreneurs themselves (Garmaise(2001)). At the same time, the entrepreneurs are engaged in unobservable effort to improve the outcomes of the business. In this paper, I study how the interaction of the two sources of asymmetries of information, namely the private expert information of the investor and the unobservable entrepreneurial effort, affects the characteristics of the optimal funding contract between venture capital investors and entrepreneurs.

The interplay between these two frictions affects the observed outcomes since empirical evidence shows that entrepreneurs react to the information they receive about the business aspects of their project from the investors. This, in turn, impacts the outcomes of the venture. I model this reaction of entrepreneurs to the information provided by the investors in the form of the effort choice of the entrepreneur. This choice of effort depends on her belief about the future profitability of the business. For instance, Howell(2021) has shown that receiving negative feedback about their business increases average venture abandonment by 13%. This can be interpreted as the entrepreneur exerting less effort in the venture. Wagner(2017) has shown that startups that received positive feedback about their business in the Chile Accelerator program fund-raised on average $15,000 more, being more likely to get funding and to survive. This can be understood as the entrepreneur working harder after the feedback to improve the outcomes for the business.

I characterize the optimal contract when investors are better informed about the project and the entrepreneur engages in a private effort by developing a static principal-agent model with an informed investor and nonverifiable entrepreneurial effort. The entrepreneur seeks to raise capital to invest in a project. This project can be either a *good* quality project or a *bad* quality project. It has a binary outcome, *success* or *failure*. The entrepreneur can engage in a costly effort to increase the probability of success, where the marginal benefit of the effort depends on the project's quality in a log-supermodular sense. Investors observe the quality of the project while the entrepreneur does not. The entrepreneur proposes a menu of contracts to the investor to screen the information about the quality of the project. The investors choose a contract out of the menu. Upon observing the type of contract accepted by the investor, the entrepreneur updates her prior belief about the quality of the project and chooses how much effort to exert. The outcome is observed, and payoffs are realized.

In a model where there is symmetric information about the project's quality, the investor is offered the lowest payment, which is just sufficient for him to break even. At the same time, the entrepreneur keeps the remaining surplus, which allows her to exert the maximum possible effort. This is the classic result of moral hazard literature (Hébert(2018), Murphy(1999)). However, when there is asymmetry of information about the quality of the project, payment to investors affects their incentives to report the quality of the project truthfully. In particular, suppose that the financial capital required from the investors is independent of the project quality. Under this environment, if the probability of success is lower for a bad quality project, then to break-even the investors in an expected sense, the entrepreneur needs to offer the investor a higher payment in case of a success, as compared to an investor observing a good quality project. This friction creates lying incentives for an investor who observes a good quality project. Thus, the entrepreneur needs to leave some information rents to this investor. This result arises in the classic model of screening in adverse selection. In this paper, these incentives interact with the unobservability of the effort of the entrepreneur.

Here, the effort choice depends on the residual return that she receives if success is realized. The lowest payment to the investor ensures that the entrepreneur exerts the highest effort given the environment. It affects the probability of success for any given project and thereby the expected payoff of the investor. This further alters the incentives of the investor to truthfully report the quality of the project.

Specifically, I show that under the assumption of a log-supermodular probability of success function, the investor always has incentives to underreport the quality of the project if the entrepreneur tries to break even with the investor. So, an investor who observes a good quality project always earns positive rent in equilibrium. This information rent is decreasing, however, in the payment promised to the investor in the bad project (through the entrepreneur's choice of effort). This force leads to the tradeoff between maximizing the surplus for the good quality and the bad quality projects. The tradeoff arises because a lower information rent for an investor observing a good project implies a higher surplus generation through a higher residual return for the entrepreneur. But this can come only at the cost of a higher promised payment to an investor observing a bad project, thereby reducing the surplus generated if it is a bad project.

The interaction of these two information frictions results in an expected payoff function for the entrepreneur that is convex in the payment promised to the investor for a bad project. In particular, even though I assume that the entrepreneur and the investor are risk-neutral to begin with, endogenous preference for risk over the payment to the investor arises for the entrepreneur. This informs us about other dimensions of the risk that the entrepreneur may prefer, other than those already established in the literature, like overoptimistic beliefs and tolerance for failures. This convexity is partly because the effect of the payment choice on effort is of second order.

Due to the convexity of the entrepreneur's expected payoff, a threshold on the prior belief about the project's quality naturally emerges . When the entrepreneur's prior belief in the project being of high quality is sufficiently low, they may prefer to maximize the surplus generated from a low-quality project, even if this entails forgoing a potentially greater surplus from a high-quality project and incurring a higher agency rent to the investor. Conversely, a sufficiently optimistic prior leads to the opposite preference. This trade-off between project types constitutes a novel mechanism through which information asymmetry regarding project quality interacts with non-contractible effort. Specifically, this interaction results in a corner solution to the entrepreneur's optimization problem, eliminating the possibility of a continuous trade-off between allocative efficiency and agency costs.

I further show that this endogenous preference for risk may not be shared by the social planner. Depending on the functional form of the probability and cost function, the social planner may either want to continuously adjust surplus maximization between both types of projects or may want to gamble over payments just like the entrepreneur. But even if the social planner chooses to gamble, the intensity of the trade-off is lower. The threshold on prior for social planner at which it substitutes the surplus of bad project for the surplus of good project is higher than the threshold on prior for the entrepreneur. It implies that the equilibrium contract is inefficient for intermediate priors where it is payoff-improving for the entrepreneur to sacrifice some surplus from the bad project to save the rent paid to the investor, but not for the social planner.

The last section of this paper explains the effect of the level of investment on the efficiency of the equilibrium contract. This effect is ambiguous and depends on the properties of the probability function and parameter values. However, given the characteristics of some industries, intervention by the government in the form of investment subsidies can be welfare improving and enhance the level of aggregate innovation in the economy.

Furthermore, entrepreneurs, in general, are proven to be overconfident (Cooper, Woo, and Dunkelberg(1988), Forbes(2005), Baron(1998), Koellinger, Minniti, and Schade(2007)). In line with this, I analyze the effect of divergence in the beliefs of the social planner and the entrepreneur on the optimal contract and its efficiency. I show that overconfidence is never welfare-improving for the entrepreneur. On the other hand, underconfindence can have ambiguous effect. In particular, severe underconfidence is bad for the entrepreneur but mild underconfidence improves the efficiency of the equilibrium contract. Given the behavior of entrepreneurs, I show that the optimistic bias in entrepreneurs' beliefs about the quality of their project is harmful to the outcome of the venture as well as to the overall welfare of the economy.

This paper is structured as follows. Section 2 discusses the related literature. Section 3 presents the model and establishes two useful benchmarks. Section 4 introduces asymmetric information and characterizes optimal contract. Section 5 describes the constrained efficient contract (second-best contract) and provides an analysis of the efficiency of the equilibrium contract. Section 6 concludes. All proofs are relegated to the Appendix.

2 Literature Review

Existing literature studies the role of information frictions on the type of contracts issued while raising investments for risky start-up businesses. Traditionally, these papers assume that the asset or the project owner (who is also the issuer of the security) has more information about the project than the investor (Leland and Pyle(1977), DeMarzo and Duffie(1999), Gorton and Pennacchi(1990), Nachman and Noe(1994), Biais and Mariotti(2005)). However, the notion that investor can have more information than the issuer or the project owner has also gained some importance.

One of the first few papers to recognize and address this information friction is Garmaise (2001), which studies a model of financing of a small-business venture in which it is presumed that outside investor have greater experience in project evaluation than the entrepreneur. The paper contrasts equilibrium financial contracts in cases where the entrepreneur does and does not acknowledge that investor possess expert information. In contrast, this paper focuses on the case where the entrepreneur does acknowledge that investor have expert information following empirical work by Howell(2021) which shows that entrepreneurs react to the negative feedback that they receive from the investor about their business prospects. There is further evidence that entrepreneurs react to the information they receive from experts. For example, Yu(2020) studies how accelerators are useful for entrepreneurs, focusing on the information they provide about the quality of the idea.

Axelson(2007) studies the security design problem of a firm when investors have private information about the firm. The firm designs a security to be sold to the informed investors in a sealed-bid auction with K investors. I focus on the environment where the uninformed entrepreneur is offering a menu of contracts to a single investor to elicit the type of project, instead of an auction among several investors. Some empirical works that have established the presence of investor expert information include Wagner(2017), Fehder and Hochberg(2014), Gonzalez-Uribe and Leatherbee(2018), Howell(2017), and Scott, Shu, and Lubynsky (2020).

On the other hand, most of the academic work on venture capital financing explains the use of certain types of securities by assuming that the entrepreneur provides value-adding effort in the business. For instance, Casamatta(2003) analyses the joint provision of effort by an entrepreneur and by an advisor to improve the productivity of an investment project. The paper shows how outside financing arises endogenously and explains why investors like venture capitalists are value-enhancing. Schmidt(2003) studies a sequential double moral hazard problem to focus on the incentive properties of convertible securities. Repullo and Suarez(2004) characterizes optimal securities with multiple investment stages and a double moral hazard problem. Hellmann(2006) similarly explains the use of convertible securities using a double moral hazard problem between the entrepreneur and the investor. Many other subsequent papers have used versions of these double moral hazard problems to show that convertible securities are optimal in venture capital markets. However, none of these papers acknowledges the expert information that the investor might have and its effect on the optimal contracts.

Peyrache and Quesada(2010) studies the optimal contract issued by an informed investor. It assumes that the informed principal provides financing to a project in which an agent exerts effort that affects the probability of success. It shows that the optimal contract for the investor is a pooling contract in which allocation of cash-flow rights and effort of entrepreneur is independent of private information of the investor. I deviate from the assumption of the investor proposing the contract. Instead, I assume that the entrepreneur is offering a menu of contracts to the informed investor to screen the information that they have. Because the entrepreneur is the owner of the project, it is reasonable to assume that the entrepreneur promises a payment to the investor upon successful completion of the project rather than the other way round. This paper focuses on understanding how the presence of investor expertise and private information affects the incentives to work for an entrepreneur in an optimal contract offered by her.

3 Model

In this section, I lay down the basic framework of the model to study the interaction of the private information of the investor with the non-verifiability of entrepreneurial effort. Following the description of the model, I study two benchmarks in detail before analyzing the equilibrium.

3.1 Setup

Consider two risk-neutral agents: an investor (he) and an entrepreneur (she). The investor has deep pockets. The entrepreneur has no endowment but has access to a risky project. This project can either be a *success* and generate a return of R or it can be a *failure* and generate zero return. The project requires two inputs: a fixed financial investment I and entrepreneurial human capital investment, effort e. Exerting effort e imposes a cost $c(e)$ on the entrepreneur, where $c'(e) \geq 0$, $c''(e) \geq 0$ and $c(0) = 0$. The quality of the project θ can be of two types $\theta \in \{\theta_g, \theta_b\}$, where $\theta = \theta_g$ corresponds to a good quality project and $\theta = \theta_b$ corresponds to a bad quality project. Notice here that a bad quality project can be associated with a more risky project and the good quality project can be associated with a relatively less risky project. Given the entrepreneur's effort e, and the project's quality θ, the probability that the project succeeds is given by $P(\theta, e)$.

Since the entrepreneur does not have the resources to invest, she can seek financing from the investor in exchange for a share of the return if the project succeeds. Formally, the entrepreneur makes an offer to the investor to pay s if the project succeeds and nothing if it fails in return for the initial investment in the project. Both agents are protected by limited liability and the opportunity cost of money is normalized to zero.

Asymmetry of information between the entrepreneur and the investor arises in two ways.

First, the quality of the project is observable only to the investors. Because the investors are more informed about the business aspects of the project, they know more about the quality of the project than the entrepreneur. Conversely, the entrepreneur has a prior μ over the project's quality where she believes that the project is good with probability μ and bad with probability $(1-\mu)$. Second, the entrepreneurial effort, which determines the likelihood of success given project's quality, is not observable to the investor. Hence, the effort is not contractible. These two sources of information asymmetry are key to my analysis in this paper. It is assumed that once an outcome is realized (either success or failure), it is perfectly observed by both the agents, namely the investor and the entrepreneur.

The timing of the model is as follows: The entrepreneur offers a menu of contracts (s_g, s_b) to the investor in exchange for the initial investment I in the project today, where s_i is the payment that the entrepreneur promises to pay to the investor in the event of a success.[1] The investor observes the quality of the project and either accepts an offer from the menu or rejects all offers. Upon observing the action of the investor (which contract is accepted or if all contracts are rejected), she updates her belief about the project's quality and decides how much effort to exert. Given the project's quality and effort choice of the entrepreneur, success takes place with probability $P(\theta, e)$ and the return R is realized. If the project is successful, the entrepreneur gets a cash flow of $(R - s_i)$ and the investor receives s_i. If the project is unsuccessful, both get a payoff of zero.

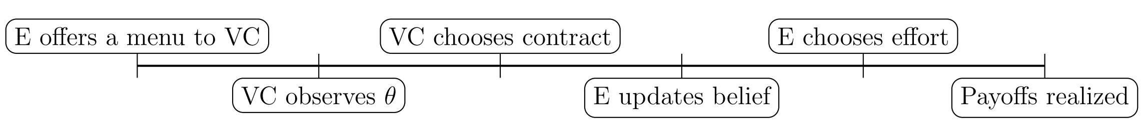

Figure 1: Timeline

Therefore, the expected utilities of the entrepreneur and the investor are given by $U(\mathbf{e}, \mathbf{s})$

[1] Assuming two offers in the menu because using the revelation principle, an equilibrium outcome can be replicated by a direct revelation mechanism

and $V(\theta, \mathbf{e}, \mathbf{s})$ respectively, where

$$U(\mathbf{e}, \mathbf{s}) = \mu[P(\theta_g, e_g)(R - s_g) - c(e_g)] + (1 - \mu)[P(\theta_b, e_b)(R - s_b) - c(e_b)] \tag{1}$$

$$V(\theta_i, e_j, s_j) = P(\theta_i, e_j)s_j - I \tag{2}$$

In this paper, I characterize a fully separating equilibrium contract. Specifically, I assume that whenever the investor is indifferent between choosing one type of contract over the other, they choose the one that reveals the project quality truthfully. Hence, in the expected payoff for the entrepreneur $U(e, s)$ (1), the entrepreneur first takes an expectation over net payoff given a project quality and then, using the belief μ, she takes an expectation over the project quality. The expected payoff of the investor (2) from a contract j , given observed project quality i is given by the $V(\theta_i, e_j, s_j)$ function net of the investment costs. Next, we impose some assumptions on the probability of success function, $P(\theta, e)$ to ensure that a non-trivial solution to the problem exists.

Assumption 1. *$P(\theta, e)$ satisfies:*

1. $P(\theta, e) \in [0, 1]$
2. $P_e(\theta, e) > 0$ *and* $P_{ee}(\theta, e) \leq 0$
3. $P(\theta_g, e) > P(\theta_b, e)$
4. $P_e(\theta_g, e)/P(\theta_g, e) > P_e(\theta_b, e)/P(\theta_b, e)$ *(Log-supermodularity)*

Assumption (1.1) ensures that the probability of success remains between zero and one. Assumption (1.2) implies that the probability of success increases as the entrepreneurial effort increases, given the project's quality and it increases at a decreasing rate. Assumption (1.3) signifies that the likelihood of success is always higher for a good quality project than a bad quality project given effort. Finally, assumption (1.4) is a log-supermodularity assumption on the probability function. This condition implies that the probability of success for a good quality project is more sensitive to changes in the effort than the bad quality project. It is analogous to the single-crossing condition in the usual adverse selection problems, and it is assumed to ensure that a separating equilibrium of the model exists. An implication of this assumption combined with the assumption (1.3) is that marginal increase in the probability of success, as effort increases, is always higher for a good quality project compared to a bad quality project, $P_e(\theta_g, e) > P_e(\theta_b, e)$, given effort e.

Next, I consider the first best social surplus. Here, the social planner wants to maximize social surplus,

$$S(e_g, e_b) \equiv \mu[P(\theta_g, e_g)R - c(e_g)] + (1-\mu)[P(\theta_b, e_b)R - c(e_b)]$$

by choosing appropriate effort choice and payment. Because the payment to the investor does not directly enter the expression for total surplus, any payment that ensures that the investor's payoff is nonnegative is optimal. The efficient level of effort choice is determined by equating the marginal benefit of the effort to the marginal cost of effort. Hence, the solution to the following simultaneous equations determines the first best optimal effort choice and payment schedule.

$$e_i^* : P_e(\theta_i, e_i)R - c'(e_i) = 0, \, \forall i \in \{g, b\} \tag{3}$$

$$s_i^* : V(\theta_i, e_i^*, s_i^*) \geq 0, \, \forall i \in \{g, b\} \tag{4}$$

The first best value of social surplus is:

$$S(e_g^*, e_b^*) = \mu[P(\theta_g, e_g^*)R - c(e_g^*)] + (1-\mu)[P(\theta_b, e_b^*)R - c(e_b^*)]$$

Assumption 2. *Both types of projects have positive NPV at first best. In other words, the parameter values are such that* $P(\theta_i, e_i^*)R - c(e_i^*) - I \geq 0, \, \forall i \in \{g, b\}$.

Assumption 2 ensures that both types of projects are profitable to begin with and the entrepreneur has incentives to undertake the project independent of the quality of the project.

3.2 Benchmarks

The environment features two frictions. First, the investor is privately informed about the project's quality θ, and second, the entrepreneur engages in an unobservable effort e. Before proceeding to the equilibrium analysis, I consider benchmarks in which I shut down each of these frictions in turn. Moreover, I will focus on analyzing separating equilibria throughout the paper.

Publicly Observable θ

First, I consider the contracts offered if the project's quality is perfectly observable by both players. In particular, I consider the case where θ is observable and e is unobservable (not contractible).

Proposition 1 (Publicly Observable θ)**.** *Suppose θ is publicly observable. In an optimal contract, the entrepreneur will extract all the surplus from the investor, i.e., $V(\theta_i, e_i, s_i) = 0$ $\forall i \in \{g, b\}$ and the optimal effort choice, e_i^u, equates the marginal benefit of effort to marginal cost of effort, conditional on the project's quality, i.e, $P_e(\theta_i, e_i^u)(R - s_i) = c'(e_i^u)$.*

Additionally, the entrepreneur chooses the minimum of s_i which satisfies the two mentioned equations simultaneously, i.e., $s_i^u = \min\{s_i : V(\theta_i, e_i^u, s_i) = 0, P_e(\theta_i, e_i^u)(R - s_i) = c'(e_i^u)\}$

See proof on page 29.

With perfect information about the project's quality, the entrepreneur extracts the entire surplus from the investor and exerts the effort which is optimal given her share in the return she expects to retain. At the first best level, it is optimal to exert effort e_i^* which equates the marginal benefit of effort in the total surplus to the marginal cost of effort. Here, however, due to the unobservability of effort, the entrepreneur chooses the effort such that the marginal cost of the effort is equal to the marginal benefit of the effort in the surplus that accrues to the entrepreneur and not the total surplus generated by the project. This leads to distortion in the effort choice compared to the first best socially optimal level.

Because the effort choice always depends on the residual return left with the entrepreneur, e_i^u is a function of s_i^u. So, the solution to this problem is a fixed point of the equation $P(\theta_i, e_i^u(s_i^u))s_i^u = I$. Parameter space can be restricted to ensure that there exists a solution to this problem. Moreover, there may be multiple fixed points that satisfy this equation. Since the entrepreneur wants to maximize her payoff, she will choose s_i which maximizes the effort choice, i.e., she will choose the minimum of the two fixed points.[2]

Proposition 2 (Constrained Efficiency: Publicly Observable θ)**.** *Suppose θ is publicly observable. The contract designed by the social planner is identical to the contract proposed by the entrepreneur in equilibrium. In other words, when θ is observable, the optimal contract is constrained efficient.*

See proof on page 31.

Intuitively, the effort choice depends negatively on the payment choice s_i to the investor. So, the social planner, like the entrepreneur, would leave zero net payoff to the investor.[3] This perfectly aligns the objective of the entrepreneur and the social planner resulting in the constrained efficiency of the equilibrium contract. It is important however to note that constrained efficient contract is not identical to the first best contract. The inherent inefficiency due to the unobservability of effort e remains even when the social planner offers the contract.

Verifiable Effort (Publicly observable e)

Next, consider the menu of contracts offered when effort is observable, and hence contractible, but the project's quality is unobservable to the entrepreneur. This is the case of no moral hazard problem for the entrepreneur, but an adverse selection problem.

[2]Additional assumptions on the probability of success function and cost of effort function guarantees that such fixed point necessarily exists. Moreover, given the restrictions on the model, there can be at most two fixed points to this problem.

[3]Although, the entrepreneur has additional incentives to leave zero rent to the investor.

Proposition 3 (Publicly Observable e)**.** *Suppose that effort is publicly observable. Then, $V(\theta_b, e_b, s_b) = 0$ and $V(\theta_g, e_g, s_g) > 0$. Moreover, e_g is at the socially optimal level, i.e., $P_e(\theta_g, e_g)R = c'(e_g)$. There is some distortion in e_b. It solves, $-\mu I\left[\frac{d}{de_b}\frac{P(\theta_g,e_b)}{P(\theta_b,e_b)}\right] + (1-\mu)[P_e(\theta_b, e_b)R - c'(e_b)] = 0$.*

See proof on page 33.

When θ is privately known to the investor and effort e is contractible, the entrepreneur designs a menu of contracts such that the investor does not have any incentives to misreport the quality of the project. So along with the participation constraint for the investor, incentive constraints are imposed to elicit truthful reporting of the project's quality from the investor. Given the assumptions on the probability of success function $P(\theta, e)$, I show in the appendix that the investor always has incentives to underreport the quality of the project and he never overreports the quality.

Log-supermodularity of the probability of success function implies that the effort choice for a bad project positively affects the incentives of the investor to underreport the project's quality and the agency rent that he can get from the entrepreneur. So, the entrepreneur balances the benefit of exerting higher effort for a bad project and the total implied cost of higher effort which is the sum of the cost of exerting higher effort and rent left to the investor. This distortion in the effort choice for a bad project is reflected in the condition mentioned in Proposition 3. Also note that the degree of this distortion is continuously increasing in μ, the prior on good quality project. This proposition implies that compared to the first best level of effort, there is no distortion at the top but there is distortion at the bottom varying positively with μ.

Proposition 4 (Constrained Inefficiency: Publicly Observable e)**.** *Suppose that effort is publicly observable. The contract designed by the social planner would always be the first best and the equilibrium contract is never constrained efficient.*

See proof on page 34.

Intuitively, the main source of inefficiency in the equilibrium contract is the rent-minimizing incentives of the entrepreneur. When the social planner designs the contract, it tries to maximize the total surplus generated in the economy and is not concerned with the distribution of the surplus between the investor and the entrepreneur. Since effort is observable, the social planner can ask the entrepreneur to exert the first best effort and can make any payment to the investor such that his participation constraint is satisfied and he has incentives to truthfully report the quality of the project. So, the rent-minimizing behavior of the entrepreneur distorts the effort choice and leads to inefficiency at the equilibrium.

Hence, when the project's quality is perfectly observed by everyone, the equilibrium contract is constrained efficient. The effort choice of the entrepreneur depends negatively on the payment made to the investor and as a result, effort is always less than its first best level. The entrepreneur makes a payment to the investor so that he breaks even in equilibrium. When the effort choice is publicly observable/contractible, the equilibrium contract is always constrained inefficient. This is due to the divergence in the goal of the entrepreneur, who wants to maximize her share of the surplus by minimizing the agency rent left to the investor, and the social planner, who wants to maximize the total surplus regardless of the distribution of the surplus. Effort choice for a good quality project is at the first best level, but effort choice for a bad quality project is less than the first best level and it is decreasing continuously in μ.

These benchmarks show that all our subsequent findings follow from the interaction of these two information frictions together.

4 Optimal Incentive Contracts

Consider the case of asymmetric information about the project's quality and unobservable entrepreneurial effort choice. The entrepreneur's objective is to design a menu of contracts that satisfies the participation constraint of the investor and ensures that the investor has incentives to truthfully report the quality of the project given that the effort choice is determined by the residual claim of the entrepreneur. In other words, the entrepreneur's objective is to maximize her expected payoff subject to the participation constraint of the investor, ex-post incentive compatibility constraint of the entrepreneur, the incentive compatibility constraint of the investor, and the limited liability constraints, i.e.,

$$
\begin{aligned}
&\max_{\mathbf{e}, s_g, s_b} && U(\mathbf{e}, \mathbf{s}) && \\
&\text{subject to} && V(\theta_i, e_i, s_i) \geq 0, \forall i \in \{g, b\} && (IR_g, IR_b) \\
& && \mathbf{e} \in \arg\max_{e'} U(\mathbf{e'}, \mathbf{s}) && (IC_e) \\
& && V(\theta_i, e_i, s_i) \geq V(\theta_i, e_j, s_j)\ \forall i, j \in \{g, b\}, i \neq j && (IC_g, IC_b) \\
& && 0 \leq s_i \leq R && \text{(Limited Liability)}
\end{aligned}
$$

In the beginning, I imposed a restriction on the probability function such that $P(\theta_g, e) > P(\theta_b, e)$. So, $V(\theta_g, e_i, s_i) > V(\theta_b, e_i, s_i)$ for any given e_i and s_i. It implies that the participation constraint for the investor who observes a good quality project automatically holds if the participation constraint for the investor who observes a bad quality project holds along with the incentive compatibility constraint of the investor observing a good quality project. Given the log-supermodularity (single crossing) assumption placed on the functional form of $P(\theta, e)$, the incentive compatibility constraint of the investor who observes a bad quality project also holds if the IR constraint for the investor observing a bad project along with the IC constraint for the investor who observes a good quality project is satisfied. Hence, the problem for the entrepreneur then reduces to,

$$
\begin{aligned}
&\max_{\mathbf{e}, s_g, s_b} && U(\mathbf{e}, \mathbf{s}) && \\
&\text{subject to} && P(\theta_b, e_b)s_b - I \geq 0 && (IR_b) \\
& && e_i \in \arg\max_{e'_i} P(\theta_i, e_i)(R - s_i) - c(e_i) && (IC_e) \\
& && P(\theta_g, e_g)s_g \geq P(\theta_g, e_b)s_b && (IC_g) \\
& && 0 \leq s_i \leq R && \text{(Limited Liability)}
\end{aligned}
$$

The incentive compatibility constraint for the investor who observes a good quality project must hold with equality in equilibrium. If IC_g is satisfied with strict inequality, then the entrepreneur can reduce s_g, increasing the effort choice e_g. The expected return on investment can increase or decrease depending on the magnitude of changes in e_g and s_g. If the payoff increases, the constraint is still satisfied. If, on the other hand, the payoff falls, then the change in s_g can be made small enough so that the constraint continues to hold. The effect of payment on the expected payoff of the entrepreneur is of first order importance and the subsequent change in the effort choice is of second order importance. So, the entrepreneur always prefers to reduce s_g to improve her payoff. The entrepreneur will continue to reduce s_g until the incentive compatibility constraint of the investor IC_g is satisfied with equality.

Furthermore, the ex-post incentive compatibility constraint for the entrepreneur (IC_e) can be replaced with the first order condition $P_e(\theta_i, e_i)(R - s_i) - c'(e_i) = 0$ because of the concavity of probability function P in effort choice and the convexity of the cost of effort function c.

Hence, the constraints on the maximization problem can be summarized as,

$$e_i : \arg\max_e P(\theta_i, e)(R - s_i) - c(e_i), \forall i \in \{g, b\} \qquad (IC_e)$$

$$s_g : P(\theta_g, e_g)s_g = P(\theta_g, e_b)s_b \qquad (IC_g)$$

$$P(\theta_b, e_b)s_b - I \geq 0 \qquad (IR_b)$$

along with the limited liability constraint. Figure 2 shows the possible values of s_b that the entrepreneur can choose so that the participation constraint of the investor who observes a bad project is satisfied.

Note that because the effort choice e_b depends on s_b, the effort choice e_g depends on s_g and s_g is determined using the equality of IC_g, the only choice variable that the entrepreneur has here is choosing the optimal payment to the investor in a bad project s_b.

Furthermore, it is not necessary that the participation constraint of the investor IR_b must be satisfied with inequality. This is because the information rent of the investor who

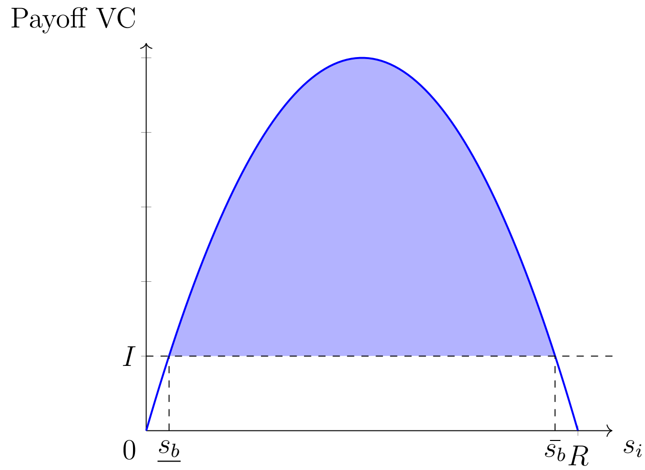

Figure 2: Participation Constraint of the Investor

observes a good quality project decreases in s_b after s_b crosses a threshold. It follows from the concavity of the expected payoff of the investor in the payment. So, while increasing s_b reduces the expected payoff from a bad project, it also reduces the information rent that the entrepreneur pays to the investor for truthfully disclosing the quality of the project. Therefore, it is possible that the entrepreneur does not want the investor who observes a good quality project to break-even.

Proposition 5 (Optimal Separating Contract)**.** *Suppose the investor is privately informed about the project's quality and the entrepreneur engages in an unobservable effort, then* $\exists$ $\bar{\mu} \in [0,1]$ *such that :*

- $\forall \mu \leq \bar{\mu}$, $\hat{s_b} = \underline{s}_b$, $\hat{e_b} = \bar{e}_b$, $\hat{s_g} = \bar{s_g}$, $\hat{e_g} = \underline{e}_g$
- $\forall \mu \geq \bar{\mu}$, $\hat{s_b} = \bar{s}_b$, $\hat{e_b} = \underline{e}_b$, $\hat{s_g} = \underline{s}_g$, $\hat{e_g} = \bar{e}_g$

where $\underline{s_b} = \min\{s_b : P(\theta_i, e_i(s_i))s_i = I\}$ *and* $\bar{s_b} = \max\{s_b : P(\theta_i, e_i(s_i))s_i = I\}$*.*

Effort choice e_g *and* e_b *are determined by the ex-post incentive constraints of the entrepreneur and* s_g *is determined endogenously by the IC constraint of the investor when project's quality is good.*

See proof on page 36.

The standard argument from moral hazard literature for choosing the minimum payment for the investor whose participation constraint matters is not applicable directly here. As usual, the payment choice (s_g, s_b) determines the ex-post effort chosen by the entrepreneur. Because the probability of success depends on the effort choice, the payments along with the effort choice determine the expected payoff of the investor. So, the payment choice directly and indirectly (through effort) affects the incentives of the investor to misreport the quality of the project.

As explained earlier in this section, the investor who observes a good quality project has incentives to underreport the quality of the project. By underreporting, he can guarantee himself a positive payoff. Hence, the entrepreneur has to leave some agency rent to the investor. Suppose now, that the entrepreneur minimizes the payment to the investor who observes a bad quality project (choosing the minimum payment that breaks even the investor) to ensure the highest possible ex-post effort level for the bad project. This will maximize the surplus for a bad project. But at the same time, it will affect the payoff that the investor who observes a good quality project receives by underreporting the quality of the project. Put simply, the expected surplus from a bad project and the agency rent of the investor varies positively with the effort choice of the entrepreneur for a bad project if the payoff of the investor who observes a bad project is held constant. So, the entrepreneur may have incentives to offer a contract that gives a slightly higher payment (s_b) to the investor in a bad project so that the resulting fall in the effort choice (e_b) might lead to a fall in the information rent of the investor $(P(\theta_g, e_b) * s_b)$.

Therefore, to begin with solving the problem for the entrepreneur, I assume that the IR_b constraint imposes restrictions on the possible solution for s_b and may not hold with equality in equilibrium. It restricts $s_b \in [\underline{s_b}, \bar{s_b}]$ where $\underline{s_b}$ is the lowest value of s_b and $\bar{s_b}$ is the highest value of s_b that satisfies the participation constraint of an investor in a bad project with equality.[4]

The Appendix shows that there is no interior solution for the separating equilibrium in this feasible region. This is primarily because the effect of payment choice on the rent left to

[4]Note that due to the dependence of effort choice on the payment, there are two solutions to the IR_b constraint holding with equality.

the investor is a first-order effect and the impact of payment on surplus through the effort choice is of second order. Hence, if the prior of the entrepreneur on the quality of the project is such that the entrepreneur wants to increase the payments to the investor who observes a bad quality project slightly to improve her share of the surplus, it is worthwhile for the entrepreneur to increase the payment to the investor at the highest possible level of the payment $\bar{s_b}$ to minimize the information rent at its lowest value.

Recall that this rent-minimizing behavior of the entrepreneur was the main source of inefficiency when e was contractible but θ was still unobservable. However, in that benchmark, the entrepreneur can continuously adjust e_b with prior μ to account for the impact of e_b on agency rent. Here, the entrepreneur can not commit to an effort choice ex-ante. Due to this lack of commitment, the payment choice determines effort choice. So, the direct and indirect effect of payment choice on the agency rent of the investor leads to this extreme result where the entrepreneur offers either the best contract for the bad project and the worst contract for the good project or vice versa. There is no continuity in the substitution between the efficiency and rent-minimizing behavior of the entrepreneur.

To understand the constrained efficiency of the equilibrium contract, it is useful to characterize the contract that will be proposed by a social planner who wants to maximize the social surplus constrained by the two information frictions.

5 Second Best Contract

Suppose there is a social planner who wants to maximize the net expected social surplus and is facing the same information and incentive constraints as the entrepreneur. The problem for this planner is the following:

$$\max_{\mathbf{e},s_g,s_b} \quad \mu[P(\theta_g,e_g)R - c(e_g)] + (1-\mu)[P(\theta_b,e_b)R - c(e_b)]$$

$$\text{subject to} \quad P(\theta_i,e_i)s_i - I \geq 0,\ \forall i \in \{g,b\} \qquad (IR_g^{SB}, IR_b^{SB})$$

$$e_i \in \arg\max_{e_i'} P(\theta_i,e_i)(R - s_i) - c(e_i) \qquad (IC_e^{SB})$$

$$P(\theta_i,e_i)s_i \geq P(\theta_i,e_j)s_j,\ \forall i,j \in \{g,b\},\ i \neq j \qquad (IC_g^{SB}, IC_b^{SB})$$

$$0 \leq s_i \leq R \qquad \text{(Limited Liability)}$$

Note again here that because $P(\theta_g, e) > P(\theta_b, e)$, an investor who observes a good project always participates if given the menu of contracts, an investor who observes a bad project always participates and the investor does not have incentives to underreport the quality of the project. Specifically, IR_g always holds if IR_b and IC_g are satisfied. Also, because of the log-supermodularity assumption on the probability of success function, IC_b holds whenever IR_b and IC_g are satisfied. Additionally, IC_g must be satisfied with equality. Although here, the social planner does not care about the distribution of surplus, it still tries to minimize s_g, because the effort choice e_g depends negatively on the payment to the investor. So, the social planner will offer s_g at its minimum possible level. However, using the argument made for the equilibrium analysis, IR_b may or may not hold with equality.

Here, the social planner does not want to minimize the rent to increase the share of the entrepreneur, but it wants to minimize s_g to improve e_g as much as possible. The social planner's objective function, hence, may or may not be convex in the payment choice s_b. Unlike for the entrepreneur, where the effect of payments on the effort choice and hence surplus was a second-order effect, here the effect of payment choices s_i on effort is of first-order importance for the social planner. This leads to the ambiguity in the shape of the objective function of the social planner.

If the objective function of the social planner is also convex in the payment s_b, then the optimal contract is described by the following proposition.

Proposition 6 (Second Best Contract)**.** *Suppose θ is private information of the investor and the entrepreneur exerts unobservable effort e. The social planner's optimal contract takes the form of a cutoff strategy.* $\exists\ \bar{\mu}^{SB} > \bar{\mu}$ *such that:*

- $\forall \mu \leq \bar{\mu}^{SB}$, $\hat{s_b}^{SB} = \underline{s}_b$, $\hat{e_b}^{SB} = \bar{e}_b$, $\hat{s_g}^{SB} = \bar{s}_g$, $\hat{e_g}^{SB} = \underline{e}_g$
- $\forall \mu \geq \bar{\mu}^{SB}$, $\hat{s_b}^{SB} = \bar{s}_b$, $\hat{e_b}^{SB} = \underline{e}_b$, $\hat{s_g}^{SB} = \underline{s}_g$, $\hat{e_g}^{SB} = \bar{e}_g$

Effort choices $\hat{e_g}^{SB}$ and $\hat{e_b}^{SB}$ are determined by the ex-post incentive constraints of the entrepreneur and $\hat{s_g}^{SB}$ is determined endogenously by the IC constraint of the investor when the project's quality is good.

See proof on page 40.

It is important to note that the threshold $\bar{\mu}^{SB}$ is strictly greater than the threshold for the entrepreneur, $\bar{\mu}$. This is because the rent minimizing incentives of the entrepreneur further increase the benefit of the offering a high s_b. These incentives are absent for the social planner. Therefore, the threshold for the social planner needs to be high enough to justify the choice of the extreme contract, which is best for the good quality project.

In the proposition below and the following sections, I characterize the efficiency of the equilibrium contract and understand its properties assuming that the social planner also has risk loving preferences over the payment schedule. The results for the interior solution to social planner's problem are presented in the appendix.

Proposition 7 (Constrained (In)Efficiency of Equilibrium)**.** *The equilibrium separating contract is constrained inefficient if $\mu \in [\bar{\mu}, \bar{\mu}^{SB}]$.*

See proof on page 42.

This follows directly from Proposition 5 and Proposition 6. The threshold cutoff on μ for the entrepreneur is always less than the threshold cutoff for a benevolent planner because the entrepreneur also takes into account the effect of higher payments to the investor in a bad quality project on the information rent left to the investor in a good quality project. For the social planner, the distribution of the surplus is immaterial, he only tries to maximize the total value of the surplus. Hence, this inefficiency in the choice of contract under some priors is because it is not profitable for the social planner to offer the highest payment to the investor in a bad quality project. After all, a higher surplus from the good project does not compensate for the lower surplus from the bad project. However, the additional gains to the entrepreneur from the reduced rent to the investor and hence a higher share for herself, it is profitable for the entrepreneur to offer the highest payment to the investor in a bad quality project.

5.1 Comparative Statics and Policy Recommendations

As discussed in the previous section, the equilibrium contract offered is inefficient for intermediate values of the prior on project's quality. Hence, while thinking about the efforts to reduce the inefficiency, it is convenient to focus on this intermediate region and how it changes when the outside environment changes.

Lemma 1. *Suppose* $\Delta = \bar{\mu}^{SB} - \bar{\mu}$. *Then,*

(i) Δ *falls unambiguously only if* $[(P(\theta_g, \bar{e}_g)R - c(\bar{e}_g)) - (P(\theta_g, \underline{e}_g)R - c(\underline{e}_g))]$ *rises and* $\left[\frac{P(\theta_g,\bar{e}_b)}{P(\theta_b,\bar{e}_b)} - \frac{P(\theta_g,\underline{e}_b)}{P(\theta_b,\underline{e}_b)}\right] I$ *falls.*

(ii) Δ *rises unambiguously only if* $[(P(\theta_g, \bar{e}_g)R - c(\bar{e}_g)) - (P(\theta_g, \underline{e}_g)R - c(\underline{e}_g))]$ *falls and* $\left[\frac{P(\theta_g,\bar{e}_b)}{P(\theta_b,\bar{e}_b)} - \frac{P(\theta_g,\underline{e}_b)}{P(\theta_b,\underline{e}_b)}\right] I$ *rises.*

This result is intuitive as I should expect the difference between the two thresholds $\bar{\mu}$ and $\bar{\mu}^{SB}$ to fall unambiguously only if the contribution of the higher effort level in a good quality project increases and the contribution of offering lower rent to the investor decreases when the entrepreneur decides which type of contract to offer. This change aligns the incentives of the entrepreneur and social planner which helps in reducing the inefficiency in the form of a lower Δ. If on the other hand, the possibility of leaving lower rent to the investor improves and the prospective gains from exerting a higher effort level reduces, then it misaligns the incentives of the entrepreneur and the social planner even more which ultimately leads to an increase in inefficiency in the form of higher Δ. This result will help in understanding the level of inefficiencies in industries that require higher investment.

Proposition 8 (Effect of increase in I)**.** *Suppose I increase. Then Δ may either increase or fall:*

(i) If conditions of Lemma 1 (i) are satisfied and the gain in the surplus due to a higher effort choice for a good project is higher than the loss in surplus due to a lower effort choice for a bad project, then Δ falls

(ii) If conditions of Lemma 1 (ii) are satisfied and the gain in the surplus due to a higher effort choice for a good project is lower than the loss in surplus due to a lower effort choice for a bad project, then Δ rises.

See proof on page 43.

An immediate implication of this result is that industries that require higher initial investment, for instance, clean energy startups, and where the level of agency rent is less sensitive to changes in investment level along with the lower gains from higher effort level for good quality project will suffer from greater inefficiency on an average as compared to industries where the initial investment required is low, like software business startups. Hence, giving some investment subsidies to the sectors that require high initial investment can help in mitigating the efficiency loss and subsequent loss in surplus generation in such industries.[5]

5.2 Biased Beliefs of the Entrepreneur

This model can be useful to make predictions about the impact of divergence between the beliefs of the entrepreneur and the true prior in the industry on the efficiency of the contract offered. In general, entrepreneurs are overconfident, meaning they are more optimistic about

[5] A similar analysis for the change in the level of efficiency as the return on the project changes is not as straightforward as the analysis for the changes in the level of investment because all the variables of interest either increase or decrease due to an increase in R. Therefore, the inefficiency in high return project may either be higher or lower depending on parameter values.

the quality of their projects than the average quality in the economy (Cooper, Woo, and Dunkelberg(1988), Forbes(2005), Baron(1998), Koellinger, Minniti, and Schade(2007))

Overconfidence can be interpreted as the entrepreneur having a higher prior on the project's quality being good as compared to the prior in the economy or the prior held by the social planner. If the level of confidence of the entrepreneur, denoted by her prior that the project is of good quality, is higher than the belief of the social planner, then I assume that the entrepreneur is overconfident. I similarly define underconfidence. When the prior of the entrepreneur on the good project is lower than the true prior, the entrepreneur is assumed to be underconfident. Suppose the prior of the entrepreneur is μ^e and the prior of the social planner is μ which is the true average quality of the project in the economy.

Proposition 9. *Overconfidence is never welfare improving. Underconfidence can be either welfare-improving or welfare-reducing depending on the prior probability of a good quality project.*

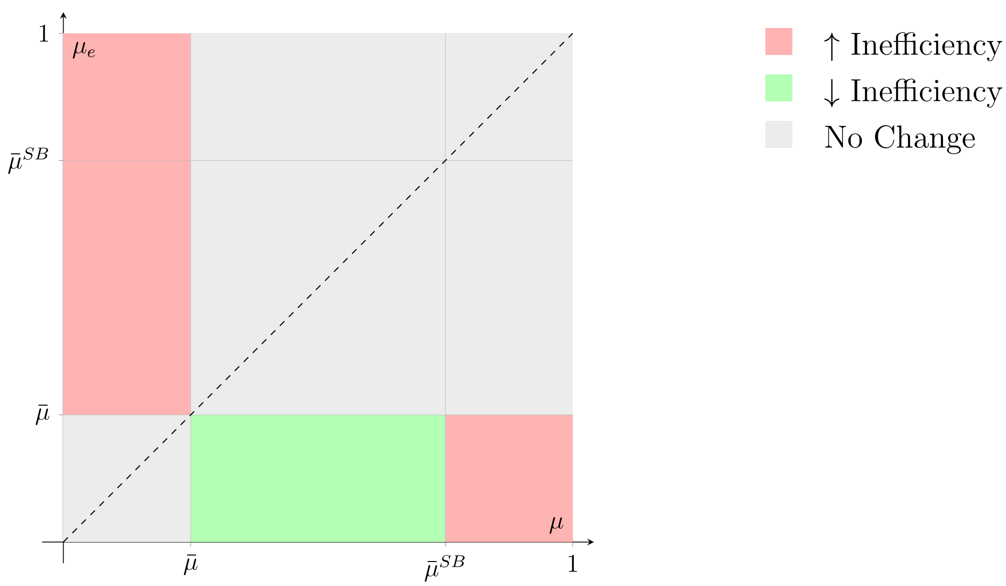

Figure 3: Impact of Biased Beliefs on Equilibrium Efficiency

Since $\bar{\mu} < \bar{\mu}^{SB}$, I can construct three intervals of priors that are of interest to me, $[0, \bar{\mu}), [\bar{\mu}, \bar{\mu}^{SB}), [\bar{\mu}^{SB}, 1]$. If the belief of the entrepreneur and the social planner lie in the same interval, i.e., μ and μ^e are in the same interval, then the presence of overconfidence would have no impact on the efficiency of the contracts offered in equilibrium. However, if $\mu \in [0, \bar{\mu})$

and $\mu^e \in [\bar{\mu}, \bar{\mu}^{SB}) \cup [\bar{\mu}^{SB}, 1]$, then if the entrepreneur was not overconfident, the equilibrium outcome would have been efficient. But now, the equilibrium outcome will be inefficient and this is purely stemming from the overconfidence of the entrepreneur. Additionally, if the project turns out to be of bad quality then the probability of this venture surviving is lower. On the other hand, if $\mu \in [\bar{\mu}, 1]$ then overconfidence does not impact the efficiency of the contract issued and the equilibrium outcome.

Therefore, in industries where the prior of having a good quality project is very low, overconfidence is detrimental to the outcome of the venture and the overall efficiency of contracts in the industries. Bad projects will fail with a higher probability (note that in industries where the prior on project's quality is very low, the number of bad quality projects is on average high), which would mean very few successful startups in these industries.

Furthermore, I comment on the effect of underconfidence on the efficiency of the equilibrium contract. Underconfidence arises when $\mu^e < \mu$. In words, the entrepreneur is more pessimistic about the project's quality than the social planner. Again, if μ and μ^e are in the same interval, then underconfidence does not impact the efficiency of the equilibrium. But if $\mu \in [\bar{\mu}, \bar{\mu}^{SB})$ and $\mu^e \in [0, \bar{\mu})$, then the entrepreneur is more pessimistic than the social planner and this can be good for the efficiency of the contract. Because the entrepreneur believes that the project probably is of bad quality, she does not want to compromise on the returns generated if the project turns out to be bad, in exchange for the benefit of paying lower rent to the investor if the project is good. This achieves the contract which is preferred by the social planner.

However, if the entrepreneur is severely underconfident, in other words, $\mu \in [\bar{\mu}^{SB}, 1]$ and $\mu^e \in [0, \bar{\mu})$, then the bias in the belief of the entrepreneur is reducing the efficiency of contracts offered in equilibrium because she is too pessimistic about the quality that she does not want to take the risk of promising higher payment to the investor in case the project turns out to be bad.

Hence, overconfidence either does not impact the efficiency of the contracts offered in equilibrium or worsens it in industries where the general prior on the project's quality is low enough. Underconfidence, on the other hand, can be either welfare-improving or welfare-reducing depending on the prior belief of the social planner (or the general belief). If it lies in the intermediate range in $[\bar{\mu}, \bar{\mu}^{SB})$, then it is welfare improving. But if it is too high $\mu \geq \bar{\mu}^{SB}$, then severe underconfidence is detrimental to welfare maximization.

6 Conclusion

In markets of innovation, venture capitalists often possess the business expertise necessary for the successful commercialization of the innovative and technical ideas of the entrepreneurs. This paper has studied the interplay of private information of liquidity providers, i.e., the investors, at the time of financing of a business with the private effort of the entrepreneur, and their impact on the equilibrium contract. The entrepreneur designs a menu of contracts to elicit truthful reporting of the project's quality and at the same time maximize her payoff. I have shown that in equilibrium, there is a stark tradeoff between surplus maximizing and rent minimizing behavior of the entrepreneur. This gives rise to a bang-bang type solution for the optimal menu.

In particular, when the prior of the entrepreneur on the good quality project is sufficiently low, the entrepreneur offers the contract that maximizes the surplus (given the constraints due to information frictions) generated by the bad quality project, compromising the high surplus production of the good quality project and paying higher agency rent to the investor. If, conversely, the prior on the good quality project is high enough, then the optimal menu of contracts changes completely. The entrepreneur offers the contract that maximizes the surplus generated by the good quality project, forgoing the high surplus production for the bad quality project. The novel result here is the contrasting nature of contracts issued as soon as the prior on project's quality crosses a threshold. There is no continuous substitution between efficiency and rent-minimizing behavior.

Furthermore, I showed that the equilibrium contract is constrained efficient for very high and very low priors, but it is inefficient for intermediate priors. In this region, it is profitable for the entrepreneur to give up a high surplus from the bad project to reduce the agency rent paid to the investor along with increasing the surplus from the good project. However, it is not profitable for the social planner to make this substitution because the payoff of the social planner, which is the total surplus generated, is independent of the distribution of the surplus produced. So, the gain by reducing the agency rent induces the entrepreneur to make the substitution but not the social planner. This is the only source of inefficiency in equilibrium

In the end, I showed that the level of inefficiency can vary positively or negatively with the initial level of investment. However, government intervention in the form of investment subsidies can be efficiency-improving in certain industries. Moreover, I demonstrated that the overconfidence of the entrepreneur is never good for the outcome of the venture and for the efficiency of the equilibrium. On the other hand, being a little conservative, i.e., mild underconfidence can be welfare-improving. However, severe underconfidence is detrimental to efficiency.

The results in this paper can be further extended to understand the impact of these information frictions on the type of security issued by the entrepreneur when there is a continuum of outcomes. It will be interesting to investigate if the knife-edge kind of contract that I get in this paper continues to persist in the security design or if the availability of multiple outcomes smoothens the trade-off between efficiency and agency rent.

Appendices

A Benchmarks

A.1 Publicly Observable θ

Proposition 1 (Publicly Observable θ)**.** *Suppose θ is publicly observable. In an optimal contract, the entrepreneur will extract all the surplus from the investor, i.e.,* $V(\theta_i, e_i, s_i) = 0$

$\forall i \in \{g, b\}$ *and the optimal effort choice,* e_i^u*, equates the marginal benefit of effort to marginal cost of effort, conditional on the project's quality, i.e,* $P_e(\theta_i, e_i^u)(R - s_i) = c'(e_i^u)$*.*

Additionally, the entrepreneur chooses the minimum of s_i *which satisfies the two mentioned equations simultaneously, i.e.,* $s_i^u = \min\{s_i : V(\theta_i, e_i^u, s_i) = 0, P_e(\theta_i, e_i^u)(R - s_i) = c'(e_i^u)\}$

Proof of Proposition 1. The problem for the entrepreneur is the following:

$$\max_{\mathbf{e}, s_g, s_b} \quad \mu[P(\theta_g, e_g)(R - s_g) - c(e_g)] + (1 - \mu)[P(\theta_b, e_b)(R - s_b) - s(e_b)]$$

$$\text{subject to} \quad P(\theta_i, e_i)s_i - I \geq 0, \forall i \in \{g, b\} \qquad (IR_b, IR_g)$$

$$e_i \in \arg\max_{e_i'} P(\theta_i, e_i)(R - s_i) - c(e_i) \qquad (IC_e)$$

$$0 \leq s_i \leq R \qquad \text{(Limited Liability)}$$

From the ex-post incentive constraint of the entrepreneur, I know that e_i is a function of s_i. Moreover, the problem of choosing the contract for type θ_b investor is separable from the problem of choosing the contract for type θ_g investor.

Because $P(\theta, e)$ is concave in e and the cost function $c(e)$ is convex in e, the ex-post effort choice for the entrepreneur is given by the solution to:

$$P_e(\theta_e, e_i)(R - s_i) - c'(e_i) = 0 \qquad (IC_e)$$

Plugging in this constraint in the objective function for the entrepreneur, I get that,

$$\max_{\mathbf{e}, s_g, s_b} \quad \mu[P(\theta_g, e_g(s_g))(R - s_g) - c(e_g(s_g))] + (1 - \mu)[P(\theta_b, e_b(s_b))(R - s_b) - s(e_b(s_g))]$$

$$\text{subject to} \quad P(\theta_i, e_i)s_i - I \geq 0, \forall i \in \{g, b\} \qquad (IR_b, IR_g)$$

$$0 \leq s_i \leq R \qquad \text{(Limited Liability)}$$

Using the solution condition for the effort choice from equation IC_e, the first-order condition for the maximization problem (unconstrained) for the entrepreneur is given by:

$$-\mu P(\theta_g, e_g(s_g)) < 0$$

$$-(1-\mu)P(\theta_b, e_b(s_b)) < 0$$

Note here that the effect of payment choice on effort choice is of second-order importance for the entrepreneur. Therefore, the entrepreneur will choose the lowest payment choice for the investor of both types. It implies that the participation constraint of the investor should hold with equality in equilibrium. In other words, the entrepreneur will extract all the surplus from the entrepreneurs, and the payment choice will be determined by the solution to the following fixed point problem:

$$P(\theta_i, e_i(s_i))s_i = I, \forall i \in \{g, b\} \tag{5}$$

I need to impose further restrictions on the functional form of the probability function $P(\theta, e)$, the cost function $c(e)$, and on the parameter values to ensure that a solution exists to this problem.

This problem can be rewritten as:

$$s_i = \frac{I}{P(\theta_i, e_i(s_i))} \equiv g(\theta_i, s_i)$$

Note that $\frac{dg}{ds_i} = \frac{-I}{P(\theta_i, e_i(s_i))^2} P_e(\theta_i, e_i) \frac{de_i}{ds_i} > 0$ since $\frac{de_i}{ds_i} < 0$.
Moreover, if $\frac{d^2 e_i}{ds_i^2} < 0$ then $\frac{d^2 g}{ds_i^2} > 0$. Otherwise, the sign of $\frac{d^2 g}{ds_i^2} > 0$ will be ambiguous and it will be difficult to make any conclusions from it. Now, a sufficient condition for $\frac{d^2 e_i}{ds_i^2} < 0$ is that $P_{eee} \leq 0$ and $c_{eee} \geq 0$, $\forall e$.

If the conditions stated above are satisfied then $g(\theta_i, s_i)$ is an increasing and convex function. Because at $s_i = 0$, $g(\theta_i, s_i) > 0$, I assume that the parameters R & I are such that $g(\theta_i, s_i)$ crosses the 45^o line.

Now, if $g(\theta_i, s_i)$ crosses the 45^o line at least once, then due to the assumptions imposed, it will cross the 45^o line exactly twice when $s_i \in [0, R]$. Because the entrepreneur wants to minimize the payments to the investor, she will choose the minimum of these fixed points.

Hence, the solution to the entrepreneur's problem in this benchmark is given by:

$$e_i : P_e(\theta_i, e_i)(R - s_i) - c'(e_i) = 0$$

$$s_i : s_i = \min\{s_i : P(\theta_i, e_i(s_i))s_i = I\}$$

For future reference, let $\underline{s_i} = \min\{s_i : P(\theta_i, e_i(s_i))s_i = I\}$ &
$\bar{s_i} = \max\{s_i : P(\theta_i, e_i(s_i))s_i = I\}$ □

Proposition 2 (Constrained Efficiency: Publicly Observable θ)**.** *Suppose θ is publicly observable. The contract designed by the social planner is identical to the contract proposed by the entrepreneur in equilibrium. In other words, when θ is observable, the optimal contract is constrained efficient.*

Proof of Proposition 2. When θ is publicly observable, then the problem for the social planner is

$$\begin{aligned} \max_{\mathbf{e}, s_g, s_b} \quad & \mu[P(\theta_g, e_g)R - c(e_g)] + (1-\mu)[P(\theta_b, e_b)R - s(e_b)] \\ \text{subject to} \quad & P(\theta_i, e_i)s_i - I \geq 0, \forall i \in \{g, b\} && (IR_b, IR_g) \\ & e_i \in \arg\max_{e_i'} P(\theta_i, e_i)(R - s_i) - c(e_i) && (IC_e) \\ & 0 \leq s_i \leq R && \text{(Limited Liability)} \end{aligned}$$

Again using the argument used in the proof for Proposition 1, ex-post effort choice for the entrepreneur is determined by the solution to equation IC_e. Plugging in this constraint in the objective function for the social planner, the maximization problem now becomes,

$$\max_{s_g, s_b} \quad \mu[P(\theta_g, e_g(s_g))R - c(e_g(s_g))] + (1-\mu)[P(\theta_b, e_b(s_b))R - s(e_b(s_b))]$$

$$\text{subject to} \quad P(\theta_i, e_i(s_i))s_i - I \geq 0, \; \forall i \in \{g, b\} \qquad (IR_b, IR_g)$$

$$0 \leq s_i \leq R \qquad \text{(Limited Liability)}$$

Here, the first order derivative of the objective function (unconstrained) with respect to the payment choices s_g & s_b is :

$$s_g : \mu[P_e(\theta_g, e_g(s_g))R - c'(e_g(s_g))]\frac{de_g}{ds_g} < 0$$

$$s_b : (1-\mu)[P_e(\theta_b, e_b(s_b))R - c'(e_b(s_b))]\frac{de_b}{ds_b} < 0$$

Hence, the social planner as well will choose the minimum payments to the investor, which was also the case when the entrepreneur designed the contracts. However, the reason for the social planner to do so is different from the reason for the entrepreneur to do so. Here, the effect of payment choice on the effort choice is a matter of first-order concern for the planner. Because the ex-post choice of effort depends negatively on the payments made to the investor, the social planner would minimize the payments to the investor. Therefore, the solution to the social planner's problem will coincide with the solution to the entrepreneur's problem, namely,

$$e_i : P_e(\theta_i, e_i)(R - s_i) - c'(e_i) = 0$$

$$s_i : s_i = \min\{s_i : P(\theta_i, e_i(s_i))s_i = I\}$$

and hence, the equilibrium in this benchmark will be constrained efficient. □

A.2 Publicly Observable e

Proposition 3 (Publicly Observable e)**.** *Suppose that effort is publicly observable. Then,* $V(\theta_b, e_b, s_b) = 0$ *and* $V(\theta_g, e_g, s_g) > 0$*. Moreover,* e_g *is at the socially optimal level, i.e.,*

$P_e(\theta_g, e_g)R = c'(e_g)$. *There is some distortion in* e_b. *It solves,* $-\mu I\left[\frac{d}{de_b}\frac{P(\theta_g,e_b)}{P(\theta_b,e_b)}\right] + (1 - \mu)[P_e(\theta_b, e_b)R - c'(e_b)] = 0$.

Proof of Proposition 3. When effort e is publicly observable and verifiable, then the problem of the entrepreneur who is proposing a separating contract to the investor is as follows:

$$
\begin{aligned}
\max_{\mathbf{e}, s_g, s_b} \quad & \mu[P(\theta_g, e_g)(R - s_g) - c(e_g)] + (1-\mu)[P(\theta_b, e_b)(R - s_b) - s(e_b)] \\
\text{subject to} \quad & P(\theta_i, e_i)s_i - I \geq 0, \ \forall i \in \{g, b\} && (IR_b, IR_g) \\
& P(\theta_b, e_b)s_b \geq P(\theta_b, e_g)s_g && (IC_b) \\
& P(\theta_g, e_g)s_g \geq P(\theta_g, e_b)s_b && (IC_g) \\
& 0 \leq s_i \leq R && \text{(Limited Liability)}
\end{aligned}
$$

Using the log-supermodularity property of the probability function and the fact $P(\theta_g, e) \geq P(\theta_b, e)$, $\forall e$, I can rewrite the maximization problem ignoring the participation constraint for the investor who observes a good quality project the incentive compatibility constraint of the investor who observes a bad quality project. All other constraints will ensure that these constraints are also satisfied. Therefore, the problem of the entrepreneur now becomes,

$$
\begin{aligned}
\max_{\mathbf{e}, s_g, s_b} \quad & \mu[P(\theta_g, e_g)(R - s_g) - c(e_g)] + (1-\mu)[P(\theta_b, e_b)(R - s_b) - s(e_b)] \\
\text{subject to} \quad & P(\theta_b, e_b)s_b - I \geq 0 && (IR_b) \\
& P(\theta_g, e_g)s_g \geq P(\theta_g, e_b)s_b && (IC_g) \\
& 0 \leq s_i \leq R && \text{(Limited Liability)}
\end{aligned}
$$

The objective function of the entrepreneur is decreasing in s_g, so the entrepreneur would prefer to set the value of s_g at its lowest possible level which satisfies the constraints as well. Hence, IC_g must bind in equilibrium. It implies that $P(\theta_g, e_g)s_g = P(\theta_g, e_b)s_b$. Plugging this constraint inside the objective function of the entrepreneur, I get that

$$\begin{aligned} \max_{\mathbf{e},s_g,s_b} \quad & \mu[P(\theta_g,e_g)R - c(e_g) - P(\theta_g,e_b)s_b] + (1-\mu)[P(\theta_b,e_b)(R-s_b) - s(e_b)] \\ \text{subject to} \quad & P(\theta_b,e_b)s_b - I \geq 0 \qquad (IR_b) \\ & 0 \leq s_i \leq R \qquad \text{(Limited Liability)} \end{aligned}$$

Now, the objective function is decreasing in s_b. This would imply that the participation constraint of the investor who observes a bad quality project must be satisfied with equality (IR_b binds). In other words, suppose that IR_b does not bind, then the entrepreneur can reduce s_b slightly, improving the payoff without violating any constraints.

The only choice variables then left are the effort choices for good and bad quality projects. After plugging in all the constraints in the objective function and taking the first-order conditions with respect to e_g and e_b, I get that:

$$e_g^o : P_e(\theta_g,e_g)R - c'(e_g) = 0$$

$$e_b^o : -\mu\frac{\partial}{\partial e_b}\left(\frac{P(\theta_g,e_b)}{P(\theta_b,e_b)}\right)I + (1-\mu)[P_e(\theta_b,e_b)R - c'(e_b)] = 0$$

Notice that e_g^o is at the first best level. However, e_b^o is less than its first best level because of the inefficiency that arises due to the unobservability of θ. Moreover, $P(\theta_g,e_g)s_g = P(\theta_g,e_b)s_b > 0$. So, an investor who observes a good quality project will always receive positive rent in equilibrium. □

Proposition 4 (Constrained Inefficiency: Publicly Observable e)**.** *Suppose that effort is publicly observable. The contract designed by the social planner would always be the first best and the equilibrium contract is never constrained efficient.*

Proof of Proposition 4. The problem of the social planner is :

$$\max_{\mathbf{e},s_g,s_b} \quad \mu[P(\theta_g,e_g)R - c(e_g)] + (1-\mu)[P(\theta_b,e_b)R - s(e_b)]$$

$$\text{subject to} \quad P(\theta_i,e_i)s_i - I \geq 0,\ \forall i \in \{g,b\} \qquad (IR_b, IR_g)$$

$$P(\theta_b,e_b)s_b \geq P(\theta_b,e_g)s_g \qquad (IC_b)$$

$$P(\theta_g,e_g)s_g \geq P(\theta_g,e_b)s_b \qquad (IC_g)$$

$$0 \leq s_i \leq R \qquad \text{(Limited Liability)}$$

Because s_g and s_b do not enter the objective function of the social planner, any s_g and s_b such that the participation constraints and the incentive compatibility constraints of the entrepreneur are satisfied can be an equilibrium contract. As far as the optimization of the objective function is concerned, because the effort choice of the entrepreneur is observable and verifiable, the social planner can implement the first best choice of effort. Mathematically, the first-order condition for maximizing the objective function with respect to the effort choice for both types of projects is the following:

$$e_g : P_e(\theta_g,e_g)R - c'(e_g) = 0$$

$$e_b : P_e(\theta_b,e_b)R - c'(e_b) = 0$$

Because the equilibrium solution for this problem entails some distortion in the choice of e_b due to the inefficiency generated by the attempt to minimize the information rent left to the investor, the equilibrium outcome is always constrained inefficient as compared to the second best effort choices under the benchmark of observable effort choice. □

B Complete Model

Proposition 5 (Optimal Separating Contract)**.** *Suppose the investor is privately informed about the project's quality and the entrepreneur engages in an unobservable effort, then* ∃

$\bar{\mu} \in [0, 1]$ *such that :*

- $\forall \mu \leq \bar{\mu}$, $\hat{s}_b = \underline{s}_b$, $\hat{e}_b = \bar{e}_b$, $\hat{s}_g = \bar{s}_g$, $\hat{e}_g = \underline{e}_g$
- $\forall \mu \geq \bar{\mu}$, $\hat{s}_b = \bar{s}_b$, $\hat{e}_b = \underline{e}_b$, $\hat{s}_g = \underline{s}_g$, $\hat{e}_g = \bar{e}_g$

where $\underline{s_b} = \min\{s_b : P(\theta_i, e_i(s_i))s_i = I\}$ *and* $\bar{s}_b = \max\{s_b : P(\theta_i, e_i(s_i))s_i = I\}$.

Effort choice e_g *and* e_b *are determined by the ex-post incentive constraints of the entrepreneur and* s_g *is determined endogenously by the IC constraint of the investor when project's quality is good.*

Proof of Proposition 5. The problem of the entrepreneur who wants to design a separating contract when the type of the investor is unobservable and the effort choice of the entrepreneur is not verifiable is the following:

$$\max_{\mathbf{e}, s_g, s_b} \quad \mu[P(\theta_g, e_g)(R - s_g) - c(e_g)] + (1 - \mu)[P(\theta_b, e_b)(R - s_b) - c(e_b)]$$

$$\text{subject to} \quad P(\theta_i, e_i)s_i - I \geq 0, \forall i \in \{g, b\} \qquad (IR_b, IR_g)$$

$$\mathbf{e} \in \arg\max_{e'} U(\mathbf{e'}, \mathbf{s}) \qquad (IC_e)$$

$$P(\theta_b, e_b)s_b \geq P(\theta_b, e_g)s_g \qquad (IC_b)$$

$$P(\theta_g, e_g)s_g \geq P(\theta_g, e_b)s_b \qquad (IC_g)$$

$$0 \leq s_i \leq R \qquad \text{(Limited Liability)}$$

Because of the log-supermodularity assumption on the probability function $P(\theta, e)$, the participation constraint of the investor who observes a bad quality project (IR_b) along with the incentive compatibility constraint of the investor who observes a good quality project (IR_g), ensures that the incentive compatibility constraint of the investor who observes a bad quality project (IC_b) is automatically satisfied. Moreover, because $P(\theta_g, e) > P9\theta_b, e)$, $\forall e$, therefore, the IR_b along with IC_g implies that the participation constraint of the investor who observes a good quality project (IR_g) is also satisfied. So, the problem of the entrepreneur then simplifies to:

$$\max_{\mathbf{e},s_g,s_b} \quad \mu[P(\theta_g,e_g)(R-s_g)-c(e_g)]+(1-\mu)[P(\theta_b,e_b)(R-s_b)-s(e_b)]$$

$$\text{subject to} \quad P(\theta_b,e_b)s_b-I\geq 0 \qquad (IR_b)$$

$$\mathbf{e}\in\arg\max_{e'} U(\mathbf{e'},\mathbf{s}) \qquad (IC_e)$$

$$P(\theta_g,e_g)s_g\geq P(\theta_g,e_b)s_b \qquad (IC_g)$$

$$0\leq s_i\leq R \qquad \text{(Limited Liability)}$$

Now, due to the concavity in effort assumption on the probability function and the convexity of the cost of effort function, the incentive compatibility constraint of the entrepreneur can be replaced with a first-order condition which is:

$$e_i : P_{e_i}(\theta_i,e_i)(R-s_i)-c'(e_i)=0 \qquad (IC_e)$$

From here, note that e_i is always decreasing in s_i. Now, IC_g must be satisfied with inequality. Suppose it is not, then s_g can be reduced slightly, which improves e_g and this together increases the payoff of the entrepreneur while still satisfying all the constraints. Hence, IC_g should be satisfied with equality. The same argument, however, can not be made for IR_b. Suppose that IR_b is not satisfied with equality, then if I try to reduce s_b so that e_b increases, the payoff of the entrepreneur improves, while still satisfying IR_b. But now, because e_b is increasing, there is no guarantee that IC_g will continue to satisfy. Hence, now I know that IC_g should bind in equilibrium and IR_b may or may not bind at this moment.

IR_b does impose some bounds on the values that s_b can take because if $s_b=0$, then it is violated, and if $s_b=R$, then also IR_b is not satisfied because $e_b=0$. The solution to the equation $P(\theta_b,e_b(s_b))s_b=I$ determines the bounds for s_b. As already shown in the proof of proposition 1, this equation has two solutions $\bar{s_b}$ and $\underline{s_b}$. So, IR_b requires that $s_b\in[\underline{s_b},\bar{s_b}]$.

Plugging in all these constraints, the altered problem of the entrepreneur is the following, where the only choice variable now left is s_b.

$$\max_{\mathbf{e},s_g,s_b} \quad \mu[P(\theta_g,e_g(s_g))(R-s_g))-c(e_g(s_g))]+(1-\mu)[P(\theta_b,e_b(s_b))(R-s_b)-s(e_b(s_b))]$$

$$\text{subject to} \quad s_b \in [\underline{s_b}, \bar{s_b}] \qquad (IR_b)$$

$$P(\theta_g,e_g)s_g = P(\theta_g,e_b)s_b \qquad (IC_g)$$

$$0 \le s_i \le R \qquad \text{(Limited Liability)}$$

Now, the first order condition with respect to s_b is the following:

$$s_b : \mu\left[\underbrace{\{P_e(\theta_g,e_g)(R-s_g)-c'(e_g)\}}_{=0}\frac{\partial e_g}{\partial s_g}-P(\theta_g,e_g)\right]\frac{\partial s_g}{\partial s_b}+(1-\mu)\left[\underbrace{\{P(\theta_b,e_b)(R-s_b)-c'(e_b)\}}_{=0}\frac{\partial e_b}{\partial s_b}-P(\theta_b,e_b)\right]$$

This simplifies to:

$$s_b : -\mu P(\theta_g,e_g)\frac{\partial s_g}{\partial s_b}-(1-\mu)P(\theta_b,e_b)$$

Note that this is not necessarily negative because for certain values of s_b, $\frac{\partial s_g}{\partial s_b} < 0$, so there is a possibility of getting an interior solution here.

Before claiming the existence of an interior solution, however, I need to check whether the altered objective function is concave in s_b or not. In other words, if the second-order condition for the maximization of the objective function holds or not. The second-order condition for the problem is:

$$\mu\left[\underbrace{\{P_{ee}(\theta_g,e_g)(R-s_g)-c''(e_g)\}}_{\le 0}\left(\frac{\partial e_g}{\partial s_g}\right)^2-\underbrace{2P_e(\theta_g,e_g)\frac{\partial e_g}{\partial s_g}}_{\le 0}\right]\left(\frac{\partial s_g}{\partial s_b}\right)^2$$

$$-\mu P(\theta_g,e_g)\frac{\partial^2 s_g}{\partial s_b^2}+(1-\mu)\left[\underbrace{\{P_{ee}(\theta_b,e_b)(R-s_b)-c''(e_b)\}}_{\le 0}-\underbrace{2P_e(\theta_b,e_b)\frac{\partial e_b}{\partial s_b}}_{\le 0}\right]$$

Using the expression for $\frac{\partial e_i}{\partial s_i}$ which is

$$\frac{\partial e_i}{\partial s_i}=\frac{P_e(\theta_i,e_i)}{P_{ee}(\theta_i,e_i)(R-s_g)-c''(e_i)}$$

I get that the SOC is always positive if $\frac{\partial^s_g}{\partial s_b^2} < 0$ which will be true given our assumption on the probability function and the cost of effort function. Hence, the objective function of the entrepreneur is convex in s_b. It implies that there is no interior solution to the maximization problem that I have.

So, the optimum for the entrepreneur's solution is at the corners. $s_b^* \in \{\underline{s_b}, \bar{s_b}\}$. The final solution will be determined by the value of the objective function at these respective corner values for s_b. Let $\bar{e_b} = e_b(\underline{s_b})$, $\underline{e_b} = e_b(\bar{s_b})$, $\bar{s_g}$ solves $P(\theta_g, e_g(\bar{s_g}))\bar{s_g} = P(\theta_g, e_b(\underline{s_b}))\underline{s_b}$ and $\underline{s_g}$ solves $P(\theta_g, e_g(\underline{s_g}))\underline{s_g} = P(\theta_g, e_b(\bar{s_b}))\bar{s_b}$ and $\bar{e_g} = e_g(\underline{s_g})$, $\underline{s_g} = e_g(\bar{s_g})$. In other words, the following are the two corresponding solutions to our variables of interest.

$$s_b = \bar{s_b}, e_b = \underline{e_b}, s_g = \underline{s_g}, e_g = \bar{e_g}$$

$$s_b = \underline{s_b}, e_b = \bar{e_b}, s_g = \bar{s_g}, e_g = \underline{e_g}$$

The value of the objective function at the first set of possible solution variables is

$$U(\underline{e_b}, \bar{s_b}, \bar{e_g}, \underline{s_g}) = \underbrace{\mu\Big[P(\theta_g, \bar{e_g})R - c(\bar{e_g})\Big]}_{\substack{\text{highest surplus} \\ \text{generated for good project}}} + (1-\mu)\underbrace{\Big[P(\theta_b, \underline{e_b})R - c(\underline{e_b})\Big]}_{\substack{\text{Lowest surplus} \\ \text{generated for bad project}}} - \underbrace{I\left((1-\mu) + \mu\frac{P(\theta_g, \underline{e_b})}{P(\theta_b, \underline{e_b})}\right)}_{\text{Lowest payoff of the investor}}$$

The value of the objective function at the second set of solution variables is:

$$U(\bar{e_b}, \underline{s_b}, \underline{e_g}, \bar{s_g}) = \underbrace{\mu\Big[P(\theta_g, \underline{e_g})R - c(\underline{e_g})\Big]}_{\substack{\text{lowest surplus} \\ \text{generated for good project}}} + (1-\mu)\underbrace{\Big[P(\theta_b, \bar{e_b})R - c(\bar{e_b})\Big]}_{\substack{\text{Highest surplus} \\ \text{generated for bad project}}} - \underbrace{I\left((1-\mu) + \mu\frac{P(\theta_g, \bar{e_b})}{P(\theta_b, \bar{e_b})}\right)}_{\text{Highest payoff of the investor}}$$

I get the highest and lowest payoff for the investor using the log-supermodularity property of the probability of success function which ensures that $\frac{P(\theta_g, e_b)}{P(\theta_b, e_b)}$ is increasing in the effort choice e_b.

Now, the optimal choice for the entrepreneur will depend on the prior on the quality of the project which is μ. Whenever μ is greater than $\bar{\mu}$ which is described as

$$\bar{\mu} \equiv \frac{[P(\theta_b, \bar{e_b})R - c'(\bar{e_b})] - [P(\theta_b, \underline{e_b})R - c'(\underline{e_b})]}{[P(\theta_b, \bar{e_b})R - c'(\bar{e_b})] - [P(\theta_b, \underline{e_b})R - c'(\underline{e_b})] + [P(\theta_g, \bar{e_g})R - c'(\bar{e_g})] - [P(\theta_g, \underline{e_g})R - c'(\underline{e_g})] + \left[\frac{P(\theta_g, \bar{e_b})}{P(\theta_b, \bar{e_b})} - \frac{P(\theta_g, \underline{e_b})}{P(\theta_b, \underline{e_b})}\right] I}$$

the optimal contract is the first set of possible solutions and if $\mu \leq \bar{\mu}$, the optimal contract is the second set of possible solutions. □

Proposition 6 (Second Best Contract)**.** *Suppose θ is private information of the investor and the entrepreneur exerts unobservable effort e. The social planner's optimal contract takes the form of a cutoff strategy. $\exists\ \bar{\mu}^{SB} > \bar{\mu}$ such that:*

- $\forall \mu \leq \bar{\mu}^{SB}$, $\hat{s_b}^{SB} = \underline{s}_b$, $\hat{e_b}^{SB} = \bar{e}_b$, $\hat{s_g}^{SB} = \bar{s_g}$, $\hat{e_g}^{SB} = \underline{e}_g$
- $\forall \mu \geq \bar{\mu}^{SB}$, $\hat{s_b}^{SB} = \bar{s}_b$, $\hat{e_b}^{SB} = \underline{e}_b$, $\hat{s_g}^{SB} = \underline{s}_g$, $\hat{e_g}^{SB} = \bar{e}_g$

Effort choices $\hat{e_g}^{SB}$ and $\hat{e_b}^{SB}$ are determined by the ex-post incentive constraints of the entrepreneur and $\hat{s_g}^{SB}$ is determined endogenously by the IC constraint of the investor when the project's quality is good.

Proof of Proposition 6. The problem for the social planner is the following:

$$\begin{aligned}
&\max_{\mathbf{e}, s_g, s_b} && \mu[P(\theta_g, e_g)R - c(e_g)] + (1-\mu)[P(\theta_b, e_b)R - c(e_b)] && \\
&\text{subject to} && P(\theta_i, e_i)s_i - I \geq 0, \ \forall i \in \{g, b\} && (IR_g^{SB}, IR_b^{SB}) \\
& && \mathbf{e} \in \arg\max_{e'} U(\mathbf{e'}, \mathbf{s}) && (IC_e^{SB}) \\
& && P(\theta_b, e_b)s_b \geq P(\theta_b, e_g)s_g && (IC_b^{SB}) \\
& && P(\theta_g, e_g)s_g \geq P(\theta_g, e_b)s_b && (IC_g^{SB}) \\
& && 0 \leq s_i \leq R && \text{(Limited Liability)}
\end{aligned}$$

Using the same argument as I made in the proof for Proposition 5, IR_g and IC_b are satisfied whenever IR_b and IC_g hold using the log-supermodularity assumption on the probability function and the property that $P(\theta_g, e) > P(\theta_b, e)$, $\forall e$. Moreover, IC_g must hold with equality. Suppose that it does not. Let $P(\theta_g, e_g)s_g > P(\theta_g, e_b)s_b$. Then the social planner can reduce s_g slightly, which will increase e_g because of the incentive compatibility constraint of the entrepreneur. This will improve the social planner's payoff without violating any constraints.

However, a similar argument can not be made for IR_b. Suppose that IR_b is not satisfied with equality, then if I try to reduce s_b so that e_b increases, the payoff of the entrepreneur improves, while still satisfying IR_b. But now, because e_b is increasing, there is no guarantee that IC_g will continue to satisfy. Hence, now I know that IC_g should bind in at the optimum and IR_b may or may not bind at this moment.

IR_b does impose some bounds on the values that s_b can take because if $s_b = 0$, then it is violated, and if $s_b = R$, then also IR_b is not satisfied because $e_b = 0$. The solution to the equation $P(\theta_b, e_b(s_b))s_b = I$ determines the bounds for s_b. As already shown in the proof of proposition 1, this equation has two solutions $\bar{s_b}$ and $\underline{s_b}$. So, IR_b requires that $s_b \in [\underline{s_b}, \bar{s_b}]$.

Plugging in all these constraints, the altered problem of the social planner is the following, where the only choice variable now left is s_b.

$$\max_{\mathbf{e}, s_g, s_b} \quad \mu[P(\theta_g, e_g(s_g))R - c(e_g(s_g))] + (1-\mu)[P(\theta_b, e_b(s_b))R - c(e_b(s_b))]$$

$$\text{subject to} \quad s_b \in [\underline{s_b}, \bar{s_b}] \qquad (IR_b^{SB})$$

$$P(\theta_g, e_g)s_g = P(\theta_g, e_b)s_b \qquad (IC_g^{SB})$$

$$0 \leq s_i \leq R \qquad \text{(Limited Liability)}$$

Now, the first order condition with respect to s_b is the following:

$$s_b : \mu\left[\underbrace{\{P_e(\theta_g, e_g)R - c'(e_g)\}}_{\geq 0} \underbrace{\frac{\partial e_g}{\partial s_g}}_{\leq 0}\right]\frac{\partial s_g}{\partial s_b} + (1-\mu)\left[\underbrace{\{P(\theta_b, e_b)R - c'(e_b)\}}_{\geq 0} \underbrace{\frac{\partial e_b}{\partial s_b}}_{\leq 0}\right]$$

Now, because the sign of $\frac{\partial s_g}{\partial s_b}$ is ambiguous, the above first-order derivative with respect to s_b

is not unambiguously negative or positive. So, there is a possibility of an interior solution. However, similar to the equilibrium model solution, the objective function is convex in s_b. Hence, there can not be an interior solution to the maximization problem of the social planner. (SHOW A PROOF HERE. YET TO DO IT). So, the optimal solution for the social planner is also a corner solution.

The two candidates for the solution are: $s_b = \underline{s_b}$ and $s_b = \bar{s_b}$. At these respective values for s_b, the value of the objective function for the social planner is:

$$U(\underline{e_b}, \bar{s_b}, \bar{e_g}, \underline{s_g}) = \mu \underbrace{\Big[P(\theta_g, \bar{e_g})R - c(\bar{e_g})\Big]}_{\substack{\text{highest surplus}\\ \text{generated for good project}}} + (1-\mu) \underbrace{\Big[P(\theta_b, \underline{e_b})R - c(\underline{e_b})\Big]}_{\substack{\text{Lowest surplus}\\ \text{generated for bad project}}}$$

$$U(\bar{e_b}, \underline{s_b}, \underline{e_g}, \bar{s_g}) = \mu \underbrace{\Big[P(\theta_g, \underline{e_g})R - c(\underline{e_g})\Big]}_{\substack{\text{lowest surplus}\\ \text{generated for good project}}} + (1-\mu) \underbrace{\Big[P(\theta_b, \bar{e_b})R - c(\bar{e_b})\Big]}_{\substack{\text{Highest surplus}\\ \text{generated for bad project}}}$$

Notice now that the trade-off is only between the surplus generated for different qualities of the project. The highest expected surplus for a bad quality project corresponds to the lowest expected surplus for a good quality project and vice versa.

So, the optimal contract for the social planner is to offer $s_b = \bar{s_b}$ if $\mu < \bar{\mu}^{SB}$ and to offer $s_b = \underline{s_b}$ if $\mu \geq \bar{\mu}^{SB}$ where

$$\bar{\mu}^{SB} \equiv \frac{[P(\theta_b, \bar{e_b})R - c'(\bar{e_b})] - [P(\theta_b, \underline{e_b})R - c'(\underline{e_b})]}{[P(\theta_b, \bar{e_b})R - c'(\bar{e_b})] - [P(\theta_b, \underline{e_b})R - c'(\underline{e_b})] + [P(\theta_g, \bar{e_g})R - c'(\bar{e_g})] - [P(\theta_g, \underline{e_g})R - c'(\underline{e_g})]}$$

□

C Inefficiency of Equilibrium

Proposition 7 (Constrained (In)Efficiency of Equilibrium)**.** *The equilibrium separating contract is constrained inefficient if* $\mu \in [\bar{\mu}, \bar{\mu}^{SB}]$.

Proof of Proposition 7. The proof of this proposition is straightforward and follows directly from Proposition 5 and Proposition 6. From Proposition 5, I know that the optimal contract is :

- $\forall \mu \leq \bar{\mu}$, $\hat{s_b} = \underline{s}_b$, $\hat{e_b} = \bar{e}_b$, $\hat{s_g} = \bar{s_g}$, $\hat{e_g} = \underline{e}_g$
- $\forall \mu \geq \bar{\mu}$, $\hat{s_b} = \bar{s}_b$, $\hat{e_b} = \underline{e}_b$, $\hat{s_g} = \underline{s}_g$, $\hat{e_g} = \bar{e}_g$

where $\bar{\mu} = \dfrac{[P(\theta_b, \bar{e_b})R - c'(\bar{e_b})] - [P(\theta_b, \underline{e_b})R - c'(\underline{e_b})]}{[P(\theta_b, \bar{e_b})R - c'(\bar{e_b})] - [P(\theta_b, \underline{e_b})R - c'(\underline{e_b})] + [P(\theta_g, \bar{e_g})R - c'(\bar{e_g})] - [P(\theta_g, \underline{e_g})R - c'(\underline{e_g})] + \left[\frac{P(\theta_g, \bar{e_b})}{P(\theta_b, \bar{e_b})} - \frac{P(\theta_g, \underline{e_b})}{P(\theta_b, \underline{e_b})}\right] I}$

And from Proposition 6, the second best contract is same except that the threshold for μ now changes to $\bar{\mu}^{SB} = \frac{[P(\theta_b,\bar{e_b})R - c'(\bar{e_b})] - [P(\theta_b,\underline{e_b})R - c'(\underline{e_b})]}{[P(\theta_b,\bar{e_b})R - c'(\bar{e_b})] - [P(\theta_b,\underline{e_b})R - c'(\underline{e_b})] + [P(\theta_g,\bar{e_g})R - c'(\bar{e_g})] - [P(\theta_g,\underline{e_g})R - c'(\underline{e_g})]}$.

$\forall \mu < \bar{\mu}$, the equilibrium contract and the second best contract is identical, i.e., offer $s_b = \underline{s_b}$. When $\mu > \bar{\mu}^{SB}$, even then the equilibrium contract and the second best contract are identical, i.e., to offer $s_b = \bar{s_b}$. It is only when $\mu \in [\bar{\mu}, \bar{\mu}^{SB}]$ that the two contracts differ, and this will be true always because $\bar{\mu} < \bar{\mu}^{SB}$. □

Proposition 8 (Effect of increase in I)**.** *Suppose I increase. Then Δ may either increase or fall:*

(i) If conditions of Lemma 1 (i) are satisfied and the gain in the surplus due to a higher effort choice for a good project is higher than the loss in surplus due to a lower effort choice for a bad project, then Δ falls

(ii) If conditions of Lemma 1 (ii) are satisfied and the gain in the surplus due to a higher effort choice for a good project is lower than the loss in surplus due to a lower effort choice for a bad project, then Δ rises.

Proof of Proposition 8. Consider first the impact of an increase in I on $\bar{e_b}$ and $\underline{e_b}$. They are determined by the participation constraint of the investor and the incentive compatibility constraint of the entrepreneur.

$$P(\theta_b, e_b)s_b = I$$

$$P_e(\theta_b, e_b)R - s_b) = c'(e_b)$$

Clearly, $\frac{\partial e_b}{\partial s_b} < 0$. Moreover,

$$\frac{\partial s_b}{\partial I} = \left(P(\theta_b, e_b) + P_e(\theta_b, e_b)\frac{\partial e_b}{\partial s_b}\right)^{-1}$$

So, at $s_b = \bar{s_b}$, $\frac{\partial s_b}{\partial I} < 0$ and at $s_b = \underline{s_b}$, $\frac{\partial s_b}{\partial I} > 0 \implies \bar{s_b}$ falls and $\underline{s_b}$ rises as I increases. This further implies that $\bar{e_b}$ falls and $\underline{e_b}$ rises with an increase in I

Now consider the numerator of $\bar{\mu}$ and $\bar{\mu}^{SB}$. Let $a = [(P(\theta_b, \bar{e_b})R - c(\bar{e_b})) - (P(\theta_b, \underline{e_b})R - c(\underline{e_b}))]$. With an increase in $\underline{e_b}$ and a fall in $\bar{e_b}$, a falls as the gap between $\underline{e_b}$ and $\bar{e_b}$ reduces.

Further, consider the impact of I on $\underline{e_g}$ and $\bar{e_g}$. Remember effort choice for good quality project is determined by the incentive compatibility constraint of the investor who observes a good quality project and the incentive compatibility constraint.

$$P(\theta_g, e_g)s_g = P(\theta_g, e_b)s_b = \underbrace{\frac{P(\theta_g, e_b)}{P(\theta_b, e_b)}I}_{\substack{\text{from the IR constraint} \\ \text{of bad type investor}}}$$

$$P_e(\theta_g, e_g)(R - s_g) = c'(e_g)$$

Again, $\frac{\partial e_g}{\partial s_g} < 0$. The rent to the investor who observes a good quality project depends non-monotonically on I. Specifically,

$$\frac{\partial}{\partial I}\left(\frac{P(\theta_g, e_b)}{P(\theta_b, e_b)}I\right) = \underbrace{\frac{\partial}{\partial e_b}\frac{P(\theta_g, e_b)}{P(\theta_b, e_b)}}_{>0} * \frac{\partial e_b}{\partial I} + \frac{P(\theta_g, e_b)}{P(\theta_b, e_b)}$$

At $e_b = \underline{e_b}$, $\frac{\partial e_b}{\partial I} > 0 \implies$ rent to the investor who observes a good quality project increase as I increase. This in turn implies that $\bar{e_g}$ falls as I increase because $\underline{s_g}$ rises.

At $e_b = \bar{e_b}$, $\frac{\partial e_b}{\partial I} < 0 \implies$ rent to the investor who observes a good quality project may either fall or rise.

Suppose first that $\frac{\partial}{\partial I}\left(\frac{P(\theta_g, e_b)}{P(\theta_b, e_b)}I\right) < 0$. Then $\bar{s_g}$ falls, as a result of which $\underline{e_g}$ rises. Let $b = [(P(\theta_g, \bar{e_g})R - c(\bar{e_g})) - (P(\theta_g, \underline{e_g})R - c(\underline{e_g}))]$. Under this case, I can see that $\frac{\partial b}{\partial I} < 0$. Now, if $\frac{\partial}{\partial I}\left(\frac{P(\theta_g, e_b)}{P(\theta_b, e_b)}I\right) > 0$ then $\bar{s_g}$ increases due to which $\underline{e_g}$ falls. Now the sign of $\frac{\partial b}{\partial I}$ is ambiguous.

It could either be positive or negative depending on which effort level $(\bar{e_g}, \underline{e_g})$ is falling more.

Finally, the last term for our analysis is $c = \left(\frac{P(\theta_g,\bar{e_b})}{P(\theta_b,\bar{e_b})} - \frac{P(\theta_g,\underline{e_b})}{P(\theta_b,\underline{e_b})}\right)I$. With an increase in I, from our analysis above and using the log-supermodularity assumption on the probability function, I know that the term inside the bracket is falling. However, the overall effect of I on c is ambiguous. Therefore, if the effect of rising I dominates, then $\frac{\partial c}{\partial I} > 0$. Otherwise, $\frac{\partial c}{\partial I} < 0$.

Now that I know the effect of I on individual terms of $\bar{\mu}$ and $\bar{\mu}^{SB}$, consider the difference between $\bar{\mu}$ and $\bar{\mu}^{SB}$.

$$\Delta \equiv \bar{\mu}^{SB} - \bar{\mu} = \frac{a}{a+b} - \frac{a}{a+b+c} = \frac{ca}{(a+b)(a+b+c)}$$

$$\frac{\partial \Delta}{\partial I} = \frac{c'(a^3 + b^2a + 2a^2b) + a'(b^2c + bc^2 - a^2c) - b'(2ac^2 + 2bca + c^2a)}{(a+b)^2(a+b+c)^2}$$

where $a' = \frac{\partial a}{\partial I}, b' = \frac{\partial b}{\partial I}$ and $c' = \frac{\partial c}{\partial I}$

(i) Consider the first situation now. I know that $a' < 0$. If $b' > 0$ and $c' < 0$ along with $b > a$, then $\frac{\partial \Delta}{\partial I} < 0$. So, the range of μ over which the equilibrium is inefficient reduces.

(ii) Now, if $b' < 0$ and $c' > 0$ along with $b < a$, then $\frac{\partial \Delta}{\partial I} > 0$. So, the range of μ over which the equilibrium is inefficient expands.

□

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